Interaction among Sciences & Technologies and the Optimum Budget Allocation

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Abstract The method for the optimum budget allocation to various sciences & technologies fields has not been studied so far much beyond the impressions of specialists. To fill this gap, this paper tries to analyze this subject rigorously.

A bisector model appropriately expressed by a 2×2 matrix shows that the long-run optimality is attained by maximizing the dominant eigenvalue of the matrix. Depending on the interdependency of those fields and on the efficacy of resource utilization of each it, either concentration or diversification is desirable. Based on this result, inferences are made on several examples of multisector models.

key words: scientific development, public policy, optimal budget allocation, sciences and technologies,

1. Introduction

Recent advances in growth theories shed some light to the mechanism of development of sciences & technologies to some extent (e.g. Romer 1986, 1990). These theories, however, paid attention to private sector investment to research & development of technologies and not to the public sector investment to more fundamental sciences & technologies.

Give that the high living standard of modern developed countries owe most to the development of scientific knowledge and those progresses are influenced by the national policies, it is natural that leading nations in the world allocate some budget directly to scientific projects and indirectly to the research institutes and universities, through which the number of specialists in each scientific field is determined, or at least influenced. Moreover, it is certain that natural resources have become increasingly less valuable compared to human capital. Economic growth in this modern world is almost exclusively determined by the scientific progresses and their spillover effects. Hence the research on the interactions among sciences and technologies is not only a problem of philosophy of science, but also a problem of economics and policy-making. We try to analyze this important subject rigorously.

This brief essay starts with the presentation of a mathematical model of interactions of sciences & technologies fields in section 1. Then, section 2 presents some analyses of the model and, based on this result, conjectures are made on multisector models. Finally, discussion and further research direction are presented in section 3 and 4.

2. The model

Sciences and technologies are interconnected in this modern society. For instance, let us take the development of computer science, which is most rapidly developing at present time. At the basis of it lies applied mathematics related to computer science. Also, progresses in material and mechanical engineering help produce smaller and more reliable semi-conductors. Since computers themselves are used for chip-design of LSI, there is some factor of self-contribution. Developments of computers contribute more or less to almost all areas of sciences and technologies through computer simulations and simplification of information processing and of writing academic papers. Likewise, development of other scientific fields helps that of some other fields and vice-versa to some extent.

Immediately these questions below come to our mind:

(1) Do we have to invest evenly to the most peripheral fields of scientific interactions, or invest intensively to the target or key fields?

(2) Do we really need big sciences such as accelerators or space development projects at this present stage, or is it more desirable to postpone these projects?

This paper considers these questions using mathematical models.

Assumptions 1

For the sake of abstraction, sciences and technologies are divided into n fields. Each of these fields is denoted by S_i, the attainment level by s_i, and their vectors $\mathbf{S}$ and $\mathbf{s}$, respectively. Technological contribution from a specific field S_j to another field S_i is denoted by r_{ij} and their matrix by $\mathbf{R} = (r_{ij})$. Strong interaction among sciences and technologies stipulates that all fields are connected at least indirectly through other fields. Let us denote budget allocations for each field by b_i.

Assuming that these numbers do not change throughout time, we write

(Equation 1) $$\frac{\partial s_i}{\partial t} = F_i(b_i, s_1, \ldots s_n, r_{i1}, \ldots r_{in}).$$

As shown in the above equation, this model does not consider the reality that the total budget increases as more technologies become available to the economy. However, it is natural to assume that more technologically influential fields contribute proportionately also to the economy, and this simplification is justifiable.

To analyze this problem further, we consider the simplest case with respect to the function.

Assumptions 2
We assume a linear approximation, that is,

(Equation 2) $F_i = G(b_i)\sum_j r_{ij} s_j$,

where the function of budgetary effect $G(b_i)$ should be monotone increasing and concave, i.e., $G'(b_i) > 0$ and $G''(b_i) \leq 0$, also $G(0) = 0$. Matrix $\mathbf{B}$ has $G(b_i)$ as its diagonal elements and zeros elsewhere. Then, equation 1 becomes $\partial \mathbf{s} / \partial t = \mathbf{BRs}$ and its solution is

(Equation 3) $\mathbf{s} = \exp(\mathbf{BR}t)\mathbf{s_0}$,

where $\mathbf{s_0}$ denote the initial value vector.

3. The optimization of a target field

Suppose that a national interest stipulates the optimization of a specific technology in the middle or long term, such as ecological protection, environmental protection, natural resource conservation, treatment of cancer or AIDS, anti-aging, military or space technologies, etc. How should we determine the budget allocation? Is it the best way to allocate most budget to the target field and cut those to others, or is it still better to allocate resources more evenly? More concretely, the former Soviet Union had put priority to military research and neglected other private sectors for a long time. Was this policy the best way to develop military technologies? Using the present model, this problem is formulated most simply as follows:

(Question 1) Suppose that the sum of budget $\sum_j b_j = b_0$ is constant, what is the best allocation of the budget to optimize the technological progress in a specific field S_i in the long run?

Concentrated investment
To maximize the very short-term development, it is obvious from Equation 2 that the budget allocation should be heavily concentrated to b_i. It is not true, however, in the long run. If we allocate all budget to field S_i and annihilate the progress of other fields, Equation 2 becomes

$$\partial s_i / \partial t = G(b_0) r_{ii} s_i$$

Hence, the absolute level of the field grows according to

$$s_i = C\exp[G(b_0)r_{ii}t], \text{ where } C \text{ is a constant.}$$

Diversified investment

Suppose that we allocate non-zero amount of the budget to each of all fields, i.e., $\forall\, i, b_i > 0$, and these do not change throughout time. Then,

$$\mathbf{s} = \exp(\mathbf{BR}t)\mathbf{s}_0 = \exp(\mathbf{BR})^t\,\mathbf{s}_0\,.$$

Non-negative matrix $\mathbf{BR}$ has a positive dominant eigenvalue by the Perron-Frobenius theorem. Denoting it by λ, the dominant eigenvalue of $\exp(\mathbf{BR})$ is $\exp(\lambda)$. The long-run growth of vector $\mathbf{s}$ converges to the corresponding eigenvector and the growth rate per unit time is given by $\exp(\lambda)$. Hence, this problem can be attributed to the budget allocation problem which maximizes the dominant vector λ of matrix $\mathbf{BR}$. It is very difficult to solve this problem in general settings.

Basic analyses and some of its inferences

Since this paper is our first attempt, we only try to attack this problem in the most simplified setting and make some inferences to some examples. Let us consider a bisector model, with which we can easily compute the eigenvalues.

$$\mathbf{B} = \begin{pmatrix} s & 0 \\ 0 & t \end{pmatrix}, \quad \mathbf{R} = \begin{pmatrix} a & b \\ c & d \end{pmatrix},$$

then,

$$\mathbf{BR} = \begin{pmatrix} s & 0 \\ 0 & t \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} sa & sb \\ tc & td \end{pmatrix},$$

and

$$\lambda = \frac{sa + td \pm \sqrt{(sa - td)^2 + 4stbc}}{2}.$$

The dominant eigenvalue is the one with the positive sign. If $bc << a,d$, i.e., the degrees of self-contribution are much larger than those of cross-contribution, then

$$\lambda \cong \frac{sa + td + |\,sa - td\,|}{2} = \mathrm{Max}(sa, td),$$

which implies that all should be allocated to the larger between a and d. In contrast, if $bc >> a,d$, i.e., the cross-contribution dominate self-contribution, then, $\lambda \cong stbc$ and $s = t = 1/2$, which implies that the perfectly diversified investment is most desirable. For a further analysis, let us assume that $s+t=1$, $a+d=1$, $bc=u$, without loss of generality. To obtain s which maximizes λ, we differentiate the above equation.

$$\frac{\partial\lambda}{\partial s}=\frac{a-d}{2}+\frac{(a+d)(sa-td)+2(1-2s)bc}{2\sqrt{(as-td)^2+4stbc}}$$

$$=\frac{2a-1}{2}+\frac{s+a-1+2(1-2s)u}{2\sqrt{(s+a-1)^2+4s(1-s)u}}=0\,.$$

Fig.1 shows the optimum s in response to a and u as two axes. Further calculation reveals that when $u\le 1/4$, the maximum λ is obtained by investing exclusively on the larger one between a and d. When $u>1/4$, it becomes optimal to diversify the budget. When $u>1$, it is always better to diversify the allocation for all a, $0<a<1$. Note that this implies a fairly tight condition that the degrees of cross-contribution are larger than those of self-contribution. More realistic assumptions that s and t should be concave functions would make diversified investment more advantageous than analyzed here.

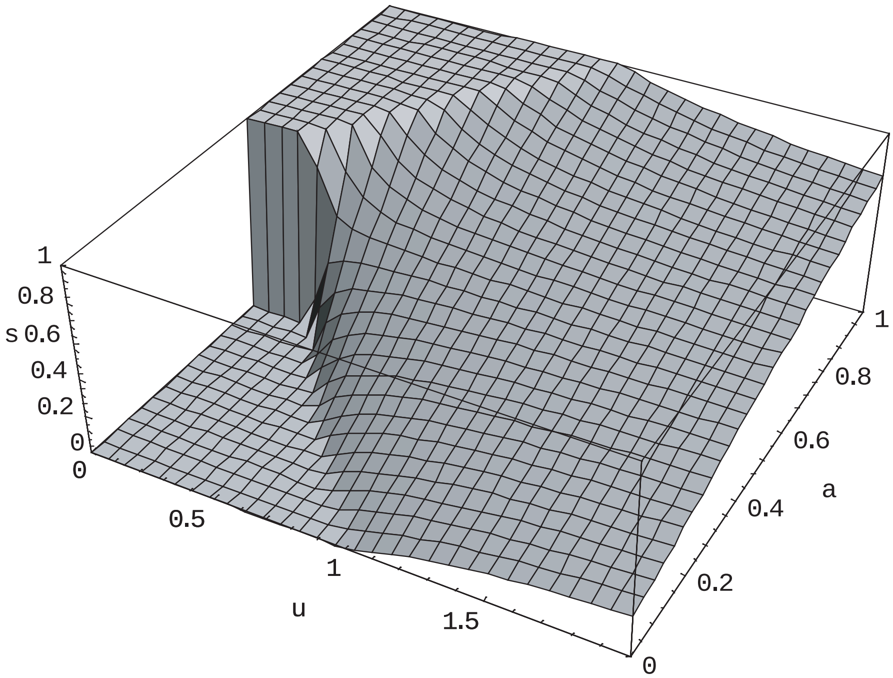


Fig.1 The value of s (the ratio of budget) which maximizes the eigenvalue with u $(=bc)$ and a

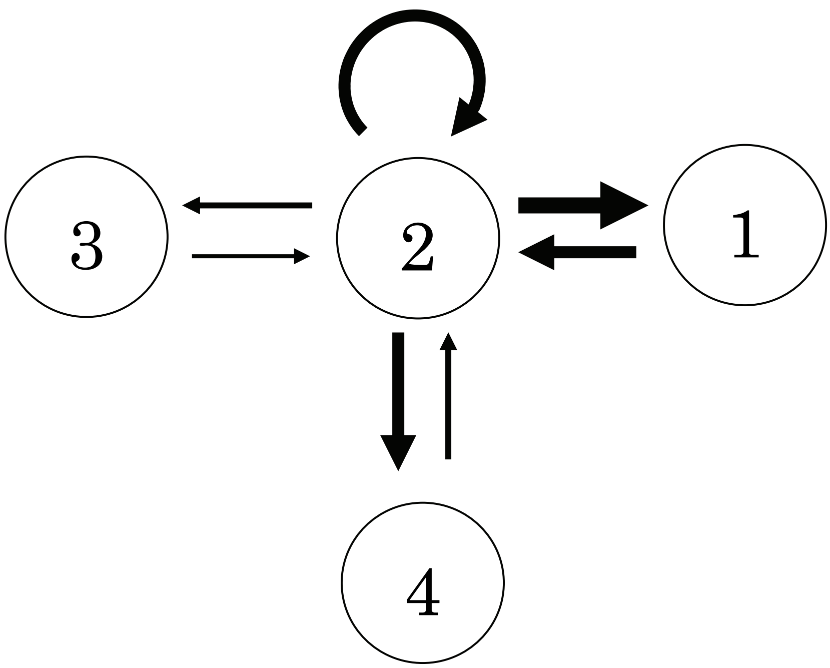
This model is based on the assumption that the matrix does not change throughout time. Although it was inevitable for the sake of analytical tractability, it in turn implies that the inferred long-run predictions form this model do not make much sense. It takes a long time for diffusion of influence from one field to the targeted field to take place, and short-term effect would be small. Hence, we should take to recommend investing on a field far from the targeted field when the matrix shows a very strong eventual spill-over effect.

Let us finally think about some typical scheme of relations among fields and conjecture the optimal policy for each case.

Example 1: there is one key field at the center and other fields are located around it like satellites (Fig.2.1)

At the center is field 2, which is relatively tightly connected to field 1. We should invest intensively to field 2 regardless of the choice of the targeted field.

Fig.2.1



Example 2: important fields are located at the edges of the scheme (Fig.2.2)

Field 1 contributes largely to other fields in spite of its location at the end of the diagram. If the targeted field is near field 1, intensive investment should be made to field 1. If it is field 4 which is far from field 1, some allocation to field 3 and 4 may well be more adequate.

Example 3: fields are divided into two relatively segregated clusters (Fig.2.3)

Fields are categorized as largely separated two groups: one consists of field 1, 2 and the other field 3, 4, 5. These are connected though field 2 and 3. Naturally, we should invest intensively to the group which include the targeted field. Also the most influential field should be allocated most.

Fig.2.2

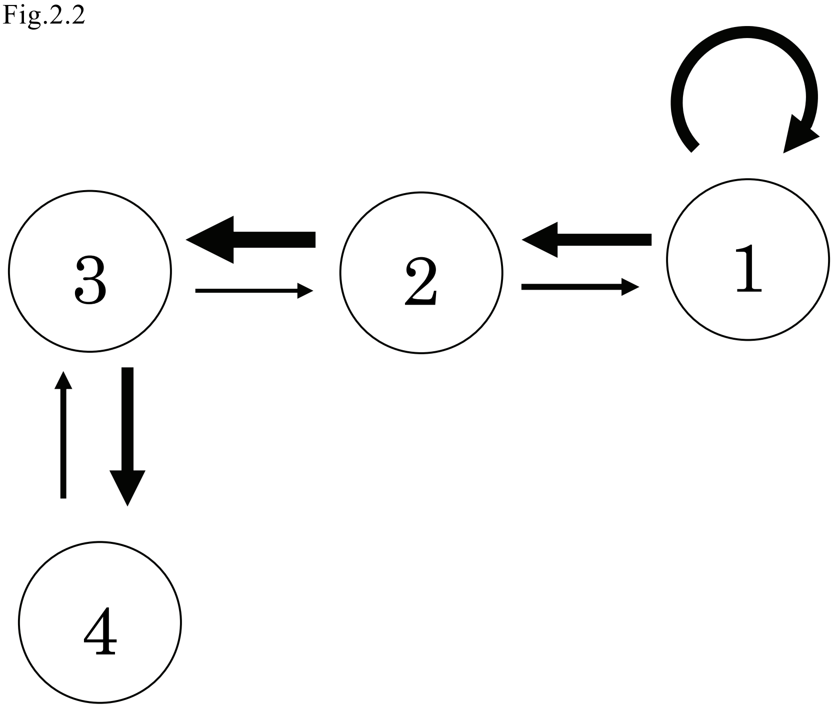
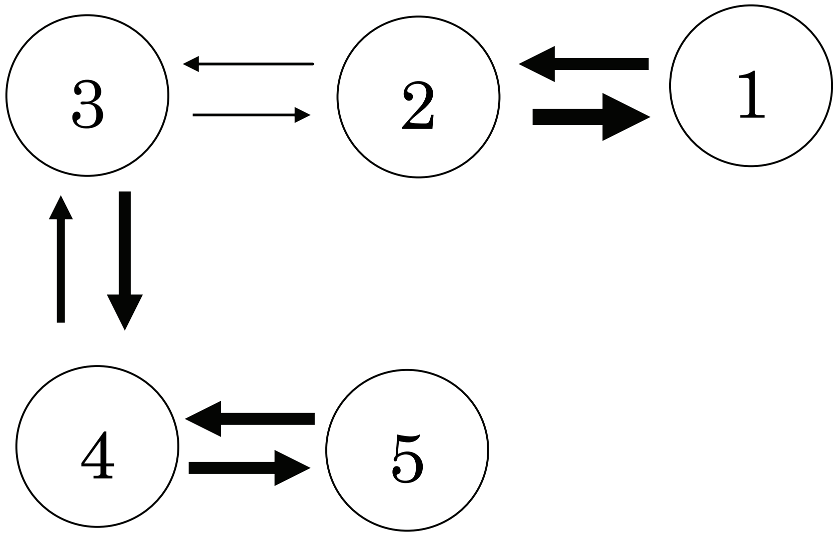


Fig.2.3



4. Discussion

As discussed before, the former Soviet Union put priority to military technologies and neglected private sector although military sector is just a peripheral or applied field from the point of view on the technological progress. These results show that this policy was a failure in the long run despite the self-evident advantage for the short-run.

Big sciences, such as a huge accelerator for the particle theories or space projects, are unlikely to make contribution not only to economic growth but also to other scientific fields. Therefore, it would have rather a negative effect on scientific progress to carry out these projects at the present stage by spending potential budget for other more promised fields. As more technologies become available in the future, any kind of goods will become cheaper to produce in comparison to the total potential production of the economy. There is no necessity at all to fulfill these big-scientific projects right now.

There may be some objections such as we can not start big-scientific projects if we only seek for spill-over effect. However, the elements of the science & technology matrix vary though the time. As stated before, this model is a first approximation of the current state of interconnection of those fields. As time goes by, the matrix will change and development of particle and space theories may come to play the key role for scientific progress. Or it may become indispensable from some political-economic stipulation in the very long run. When these happen, it will not be too late to start investing to those projects thereafter. Even in this case, private firms such as

East India Limited Company would be more efficient to do the job than bureaucratic nation states. In a capitalistic society, economic necessity itself will gather private investment correspondingly even though it is not a perfectly reasonable amount. It is certainly not a wise or rational decision to make unnecessary investment to big sciences for the sake of pursuit of "dreams of mankind" or under the sentiment of "romanticism".

5. Further research direction

In spite of the simplicity of the model, the analysis presented here is valuable because there have not been similar ones. Further research should be along three directions.

The first is the study of more specific questions based on the model, such as "Do the research carried out by firms maximizing short-term profit lead to the socially optimal development of long run levels of sciences and technologies?

The second is the enhancement of the model. The model only considers the interaction among scientific knowledge and technologies. It would be interesting to expand the model to include the economic contribution of sciences and technologies, or to unify the model with the Leontief type input-output matrix of an economy (e.g., Leontief 1986).

The third is the positive aspect of the model. In the model, we assumed the functional forms and the degrees of various contributions for computational simplicity. What are their more realistic formulations? How can we investigate interactions for actual policy prescriptions? As for this point, the Delphi method is widely used to predict the time length before various technologies become available. We may well employ the same method and ask some specialists in each field about the influences from other fields.

The importance of this analysis is well beyond a mere historical and sociological problem between economics and science & technologies. It is the very key issue for science and technologies in that it helps accelerate the development of them (which in turn would imply that exactly this research should be allocated a large amount of budget!).

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